Switching macroeconomic growth and volatility: Evidence from a Mean-Variance Markov-Switching Dynamic Factor Model[1]

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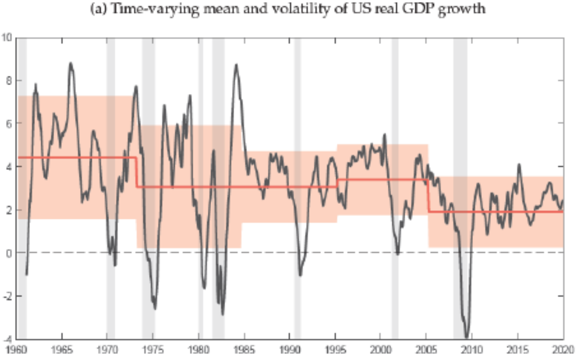
Seminar at Keio University
Tokyo, 28 Oct. 2025

[1] The views expressed here are those of the authors and do not necessarily reflect those of the OECD

Motivations

- Advanced economies have been recently hit by a sequence of major shocks: Great Financial Crisis, Covid-19, Energy crisis ...
- Main stylized facts for those economies before those shocks:
  - Long term decline in GDP growth ("Secular Stagnation")
  - Low volatility in macroeconomic series ("Great Moderation").
- Objectives of the paper:
  - Build a model which can take into account both secular stagnation, and changes in mean and variance linked to those recent shocks
  - Address the issues of turning-point analysis and GDP nowcasting, both in-sample and in real-time
  - Use a parsimonious framework

Motivations



Mean (red line) and std dev (shaded) of annualized US GDP growth rate over subsamples.

Source : Antolin-Diaz et al. (2021)

General framework

- We put forward a small-scale Extended Markov-Switching Dynamic Factor Model (MS-DFM) for the US accounting for:
  1. Co-movement in macro series
  2. Different growth rates during expansions and recessions
  3. **Variations in long-term GDP growth rate**
  4. **Time-varying volatility**
- In our model, means and variances are related to latent Markov switching processes $\rightarrow$ Mean-Variance MS-DFM: **MV-MS-DFM**:
- We estimate the model using bayesian techniques
- We provide both *in-sample analysis* and *real-time assessment* of our MV-MS-DFM for **TP detection**.
- We provide real-time **density forecasts**.

Main results

1. Evidence of a gain in goodness-of-fit from using our **MV-MS-DFM** wrt linear DFM: it accounts for a switch in variance during the *Great Moderation*
2. Match the NBER dating of turning points but send a real-time signal in advance with a several months lead
3. The *Great Recession* doesn't bring the *Great Moderation* to an end
4. Loss of about 1pp in long-run GDP growth since 2000, about 0.5pp since the *Great Recession*
5. Real-time density forecasts show that MV-MS-DFM is able to capture Growth-at-Risk features, in line with Adrian et al. (2019), but without any financial variable in the data
6. Covid period is included in the sample via specific treatments

Outline of the presentation

1. Related literature
2. Model specification
3. Estimation and data
4. In-sample results
5. Real-time out-of-sample results
6. Introduction of financial conditions
7. How to deal with Covid 19
8. Conclusion

Related literature (1/2)

- MS-DFM: Small-scale DFM with changes in growth regimes (Diebold and Rudebusch (ReStat, 1996), Kim and Nelson (ReStat, 1998))
- Univariate MS with both changes on growth and volatility regimes (McConnell and Perez-Quiros (AER, 2000), Bai and Wang (JAE, 2011))
- Integration of time-varying trend in GDP:
  - Eo and Kim (ReStat, 2016): Univariate case with MS changes in growth
  - Giordani, Kohn and van Dijk (JoE, 2007): Univariate case with both MS changes on growth and volatility regimes
  - Antolin-Diaz, Drechsel and Petrella (ReStat, 2017): Multivariate DFM with Stochastic Volatility (but no MS changes in growth)

Related literature (2/2)

	Multivariate framework (factor model)	Markov-Switching Intercept	Markov-Switching volatility of shocks	Stochastic volatility	Time-varying long-term GDP growth rate
Antolin-Diaz *et al.* (2017)	X			X	X
Bai and Wang (2011)		X	X		
Diebold and Rudebusch (1996)	X	X			
Eo and Kim (2016)		X		X	X
Giordani *et al.* (2007)[3]		X	X		X
Kim and Nelson (1998)	X	X			
Marcellino *et al.* (2016)	X			X	
McConnell and Perez-Quiros (2000)		X	X		
This paper	**X**	**X**	**X**		**X**

Model specification (1/6)

- We observe n (small) macroeconomic variables y_{it}, $i = 1, \dots, n$, supposed to depend on unobserved latent variable (common factor).
- Each y_{it} is the sum of two uncorrelated terms : one depending on the common factor and an idiosyncratic term
- **Measurement equation**

$$\Delta y_{it} = a_{it} + \gamma_i(L)\Delta c_t + u_{it}$$

$$\text{where: } \begin{cases} \Delta y_{it} : \text{demeaned growth rate of variable } i \\ a_{it} : \text{deviation from mean growth rate} \\ \Delta c_t : \text{unobserved common factor} \end{cases}$$

Model specification (2/6)

- **Underlying dynamics**:
  - (a_{it}) is a RW which captures potential low-frequency fluctuations in mean growth rates, such as the slow decline in the GDP growth rate:

$$a_{it} = a_{i,t-1} + \sigma_{a_i}.\nu_t^{a_i} \qquad \nu_t^{a_i} \sim \mathcal{N}(0,1)$$

  - (Δc_t) is a MS-AR process, with switches in mean and variance:

$$\Phi(L)\Delta c_t = \mu_{S_t,V_t} + \sqrt{1 + hV_t}.\sigma_c.\nu_t^c \qquad \nu_t^c \sim \mathcal{N}(0,1)$$

  - (u_{it}) is an AR process for any $i = 1, \ldots n$:

$$\Psi_i(L)u_{it} = \sigma_i.\varepsilon_{it} \qquad \varepsilon_{it} \sim \mathcal{N}(0,1)$$

  - $\mathrm{d}^{\mathrm{o}}(\Phi) = \mathrm{d}^{\mathrm{o}}(\Psi_i) = 1$

Model specification (3/6)

- S_t and V_t are 2 independent Markov chains with 2 regimes:

$$\begin{cases} S_t = 0 : \textit{economic recession}; & S_t = 1 : \textit{economic expansion} \\ V_t = 0 : \textit{low volatility} \quad ; & V_t = 1 : \textit{high volatility} \end{cases}$$

and

$$\begin{cases} P(S_t = 0|S_{t-1} = 0) = p_{00}\,; & P(S_t = 1|S_{t-1} = 1) = p_{11} \\ P(V_t = 0|V_{t-1} = 0) = q_{00}; & P(V_t = 1|V_{t-1} = 1) = q_{11} \end{cases}$$

- Variance of the factor's innovation:

$$\begin{cases} \textit{Variance} = \sigma_c^2 \quad ; & \text{in the low volatility regime} \\ \textit{Variance} = (1+h)\sigma_c^2; & \text{in the high volatility regime}(h > 0) \end{cases}$$

Model specification (4/6)

- Intercept of the factor's equation:

$$\mu_{S_t,V_t} = \mu_{00} + \mu_{01} V_t + \mu_{10} S_t + \mu_{11} S_t V_t$$

with

$$\begin{cases} \mu_{00} & = & \text{recession regime/low volatility} \\ \mu_{00} + \color{red}{\mu_{01}} & = & \text{recession regime/high volatility} \\ \mu_{00} + \color{blue}{\mu_{10}} & = & \text{expansion regime/low volatility} \\ \mu_{00} + \color{red}{\mu_{01} +} \color{blue}{\mu_{10} + \mu_{11}} & = & \text{expansion regime/high volatility} \end{cases}$$

Model specification (5/6)

- We include both monthly variables and a quarterly variable (GDP) in the model.
- We use the classical approximation of quarterly GDP as a weighted average of current and past monthly (unobserved) GDP values.

$$\begin{aligned}
\Delta y^q_{1t} = & \quad \frac{1}{3}a^q_{1,t} + \frac{2}{3}a^q_{1,t-1} + a^q_{1,t-2} + \frac{2}{3}a^q_{1,t-3} + \frac{1}{3}a^q_{1,t-4} \\
+ & \quad \gamma^q_1\left(\frac{1}{3}\Delta c_t + \frac{2}{3}\Delta c_{t-1} + \Delta c_{t-2} + \frac{2}{3}\Delta c_{t-3} + \frac{1}{3}\Delta c_{t-4}\right) \\
+ & \quad \frac{1}{3}u^q_{1,t} + \frac{2}{3}u^q_{1,t-1} + u^q_{1,t-2} + \frac{2}{3}u^q_{1,t-3} + \frac{1}{3}u^q_{1,t-4}
\end{aligned}$$

- For other variables: $a^m_{jt} = 0$ and

$$\Delta y^m_{jt} = \gamma^m_j(L)\Delta c_t + u^m_{jt}$$

with $\mathrm{d}^{\mathrm{o}}(\gamma^m_j) = 0 \text{ for } m \leq 3 \text{ and } \mathrm{d}^{\mathrm{o}}(\gamma^m_4) = 3$ (employment can be lagging)

Model specification (6/6): State-space representation

- The model can be cast into a state-space form
- The state vector contains $a_{1t}, \Delta c_t$ and their lags.
- Using a standard approach, measurement equations of the monthly variables, $j = 1, \ldots, 4$, are pre-multiplied by the corresponding idiosyncratic lag polynomial:

$$\begin{aligned} \Psi_j^m(L)\Delta y_{jt}^m &= \gamma_j^m(L)\Psi_j^m(L)\Delta c_t + \sigma_j^m.\varepsilon_{jt}^m, \qquad \varepsilon_{jt}^m \sim \mathcal{N}(0,1) \\ \Leftrightarrow \Delta y_{jt}^{m*} &= \gamma_j^{m*}(L)\Delta c_t + \sigma_j^m.\varepsilon_{jt}^m, \qquad \varepsilon_{jt}^m \sim \mathcal{N}(0,1) \end{aligned}$$

Allows to reduce the dimension of the state vector.

Bayesian estimation strategy

- Gibbs sampling with consecutive steps to draw the underlying state vector, the Markov variables trajectories $\widetilde{S_{1\ldots T}} = \{S_1, \ldots, S_T\}$ and $\widetilde{V_{1\ldots T}} = \{V_1, \ldots, V_T\}$, and the other constant model parameters (loading coefficients for the state vector, autoregressive parameters. variances, etc.).
- Main advantages of the Bayesian estimation: it is modular and adaptable, and it simplifies the inference on $\widetilde{S_{1\ldots T}}$ and $\widetilde{V_{1\ldots T}}$ as the monthly factor can be considered as an observed variable in the corresponding Gibbs sampling steps.
- Draws from the state vector (which includes the time-varying GDP growth rate): sequential Kalman filter/smoother with diffuse initialisation [Koopman and Durbin (2000, 2003)], then simulation smoother introduced by Durbin and Koopman (2002).

Data

- Focus on the US
- Sample: 1970m01-2019m12 (see Covid extension)
- Real GDP (QNA) from Fred Database
- The 4 monthy macro variables considered by the NBER BCDC from the Fred-MD database:
    1. Industrial production index
    2. Real manufacturing trade and sales
    3. Real personal income excluding transfer payments
    4. Non-farm payroll employment
- Vintages of monthly data are available from August 1999 onwards

Estimation results

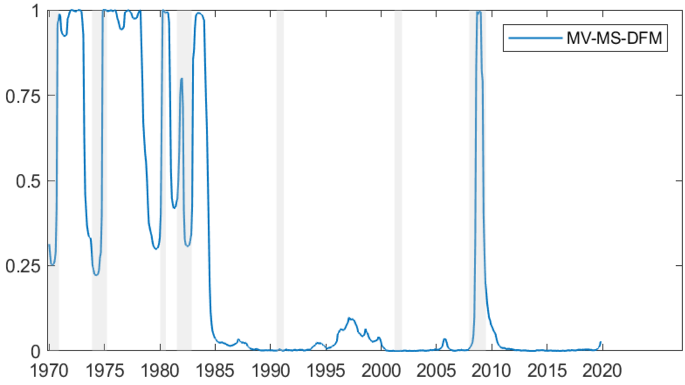
- We assume that only GDP presents a time-varying trend captured by $a^q_{1,t}$ ($a^m_{j,t} = 0$ for $j = 1, \ldots, 4$)
- Parameter estimation:

	$\hat{\mu}_{S_t,V_t}$	95% CI
Recession / High Volat	-0.77	[-1.07, -0.42]
Recession / Low Volat	-0.25	[-0.39, -0.09]
Expansion / Low Volat	0.03	[0.00, 0.06]
Expansion / High Volat	0.23	[0.08, 0.37]

- **Stronger impact of volatility during recessions** than during expansions (in line with Adrian, Boyarchenko, Giannone, AER, 2019)

Probability of being in high-volatility regime

- The *Great Recession* doesn't imply the end of the *Great Moderation*

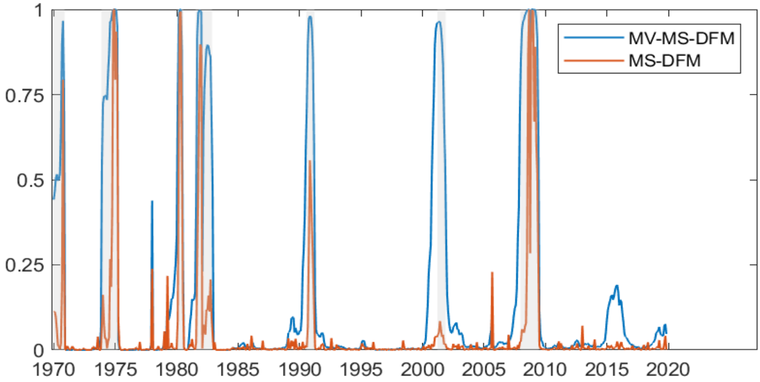


Estimation sample: 1970M01-2019M11. Data vintage: 2019M12.

Shaded areas correspond to NBER recessions.

Probability of being in recession

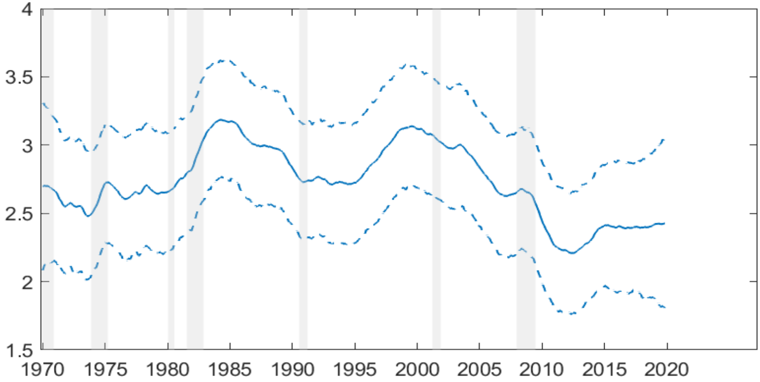
- Switches in macro volatility help to capture recessions during the *Great Moderation*



Smoothed probability of being in recession stemming from the MV-MS-DFM (blue) and from the Intercept only MS-DFM (red)

Long-term decline in US GDP growth

- Loss in long-run GDP growth is about 1pp since 2000 (about 0.5pp since the *Great Recession*)



Smoothed estimate of the US long-run GDP growth rate.
Estimation sample: 1970M01-2019M11. Data vintage: 2019M12.

Ranking of various models: MLL and DIC

Two standard tools for model comparison in a Bayesian framework.

1. *Marginal Log Likelihood* (MLL): $\log f(\widetilde{\Delta y_{1\ldots T}})$.
   - If $\tilde{\theta}^*$ is the posterior mean of θ, the MLL is computed as
     $$\log f(\widetilde{\Delta y_{1\ldots T}}) = \log f(\widetilde{\Delta y_{1\ldots T}}|\tilde{\theta}^*) + \log \psi(\tilde{\theta}^*) - \log \psi(\tilde{\theta}^*|\widetilde{\Delta y_{1\ldots T}})$$
   - Model with the highest MLL is preferred

2. *Deviance Information Criterion* (DIC):
   - If $\tilde{\theta}$ is the vector of model parameters, and $\tilde{\theta}^*$ the posterior mean of θ:
     $$\begin{aligned} DIC = & \quad \{E_{\theta|Y}(-2\log f(\widetilde{\Delta y_{1\ldots T}}|\tilde{\theta}))\} \\ + & \quad \{E_{\theta|Y}(-2\log f(\widetilde{\Delta y_{1\ldots T}}|\tilde{\theta})) + 2\log f(\widetilde{\Delta y_{1\ldots T}}|\tilde{\theta}^*))\} \end{aligned}$$
   - The 1st term decreases when the fit improves.
     The 2nd term is a penalty for model complexity.
   - Model with the lowest DIC is preferred

Ranking of various models: Results

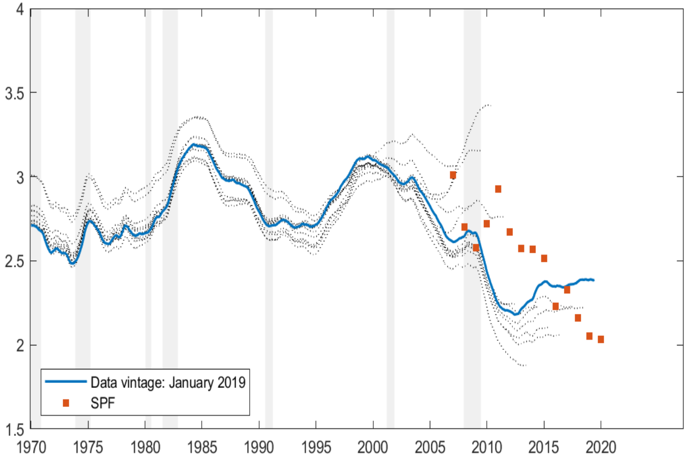
- DIC and MLL computations:

Specification	DIC	MLL
Linear DFM	3883.1	-1978.3
MS-DFM: MS intercept only	3822.2	-1963.4
MS-DFM: MS intercept & volat	**3720.9**	**-1934.7**
MS-DFM: MS intercept & volat+ TV trend	3728.6	-1937.4

- Both MS features, especially MS on volatility, improve DIC and MLL measures as compared to a linear DFM
- Best specification is 3
- 4 doesn't improve in sample fit, but it is very close in terms of DIC and ML (focusing on GDP forecasting performance will lead to a different conclusion)

Real-time estimation of long-term US GDP growth

- Smoothed estimates of the US annual long term GDP growth rate based on real time vintages (2007m1-2019m11):



Real-time detection of turning points

- Identification of peaks and troughs using a simple rule based on a threshold for the probability of recession:
  - announce a recession when this probability moves from below to above 0.7 and stays above 0.7 for 3 months: the recession start is the 1st month for which it is > 0.5
  - announce the end of a recession when this probability moves from above to below 0.3 and stays below 0.3 for 3 months: the recession end is the last month for which it is > 0.5
- Dates of peaks and troughs are in line with NBER Business Cycle Dating Committee
- A lead in announcement dates (only 2 events)

Peak date		**Announcement date**		**Difference**
NBER BCDC	MV-MS-DFM	NBER BCDC	MV-MS-DFM	
2007m12	2007m11	2008m12	2008m10	**-2**

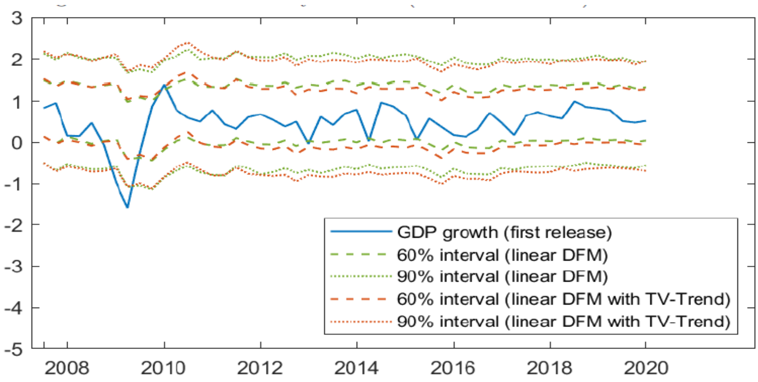
Trough date		**Announcement date**		**Difference**
NBER BCDC	MV-MS-DFM	NBER BCDC	MV-MS-DFM	
2009m06	2009m06	2010m09	2009m10	**-11**

Real-time density forecasts

- Conditional forecast distribution of GDP for the current and the next quarter Q can be computed in real-time at the end of each month of $Q-1$ and Q, using the measurement equation (conditionally to the contemporaneous values of the 4 monthly variables).
- Here we focus on the 6-months horizon forecast: made during the 1st month of $Q-1$ (to be compared with the 1st release of GDP, produced in the 1st month of $Q+1$).
- A sequence of forecasts is obtained for each iteration of the Gibbs sampler, and those forecasts are averaged across draws.
- Density forecasts incorporating all sources of uncertainty can be computed.

Real-time density forecasts

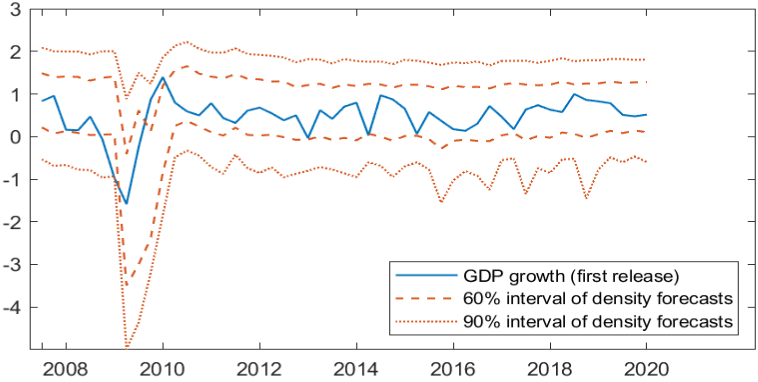
Figure: Real-time density forecast at a 6-months horizon: DFMs



Expanding estimation sample starting 1970M01. Forecast density for each quarter is based on 7500 draws of the Gibbs sampler, with the first 2500 discarded.

Real-time density forecasts

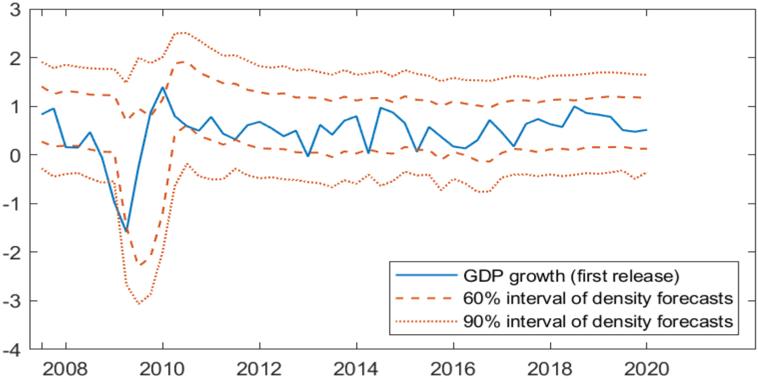
Figure: Real-time density forecast at a 6-months horizon: DFM+MS in mean



Expanding estimation sample starting 1970M01. Forecast density for each quarter is based on 7500 draws of the Gibbs sampler, with the first 2500 discarded.

Real-time density forecasts

Figure: Real-time density forecast at a 6-months horizon: MV-MS-DFM



Expanding estimation sample starting 1970M01. Forecast density for each quarter is based on 7500 draws of the Gibbs sampler, with the first 2500 discarded.

Real-time density forecasts

Figure: Quantile Scores

Table 5: Relative predictive density accuracy over 2007Q2–2019Q4, compared to the MV-MS-DFM

	5% quantile score	10% quantile score	qw-CRPS-L
Linear DFM	0.86*	0.84	0.98
Linear DFM with TV-Trend	0.82*	0.79**	0.95
MS-DFM where only the mean growth rate can switch	0.87	0.83***	0.95**

*Note: A ratio below 1 indicates that the predictive density of the MV-MS-DFM is more accurate than the alternative. *, ** and *** indicate the statistical significance of differences in quantile scores and continuous ranked probability scores at the 10%, 5% and 1% levels, respectively. While limiting distributions in Gneiting and Ranjan (2011) are obtained for fixed-length estimation samples, their relative accuracy test is here applied to estimation samples expanding from 1970-2007 to 1970-2019.*

Real-time density forecasts

- Main features:
  - Great Recession: lower average forecast + larger forecast uncertainty.
  - Downward shift of the left-hand side of the forecast distribution.
  - Upper part of the forecast density remains approx. unchanged.
- $\rightarrow$ In line with Adrian et al. (2019) results.
- But our results are obtained:
  - without any financial variable
  - with a MV-MS approach and not a quantile regression approach
  - taking advantage of the Gibbs sampler estimation to build density forecasts from a MS-DFM, which hadn't been done before (first version of the paper, Jan 2020).
- The results of our real-time density forecasts shows that the interest of MS-DFM, or MV-MS-DFM, goes beyond the evaluation of the probability to be in recession.

Introduction of financial conditions

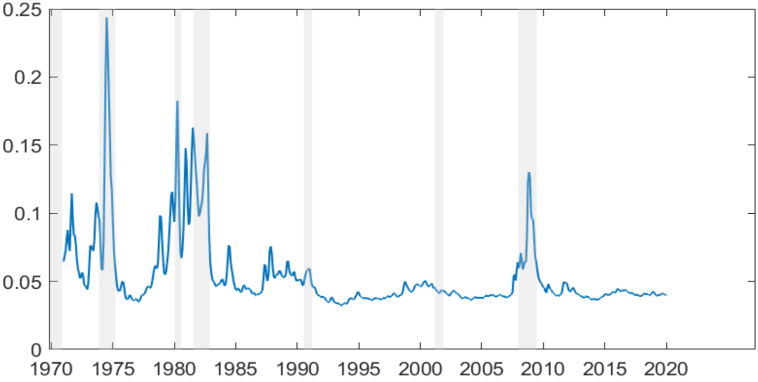
- To build on the link with Adrian et al. (2019), we introduce financial conditions via the Chicago FED's NFCI index.
- Two ways of introducing financial conditions:
  1. add the NFCI to our dataset and estimate the same models (MV-MS DFM and MS-DFM)
  2. use a probit specification: the proba to switch for a state to another depends on the current state of the economy and on the NFCI (in line with Filardo and Gordon (1998) or Caldara et al. (2021)).
- Here: results obtained with MV-MS-DFM without switches in variance

Introduction of financial conditions

- Using (1): the NFCI is only very weekly related to the common factor (posterior distribution of γ_5^m centered around 0 with limited dispersion).
- Using (2), the results are the following:
    - With a probit specification, the NFCI significantly contributes to increase the probability of falling or staying in recession.
    - However, based on the quantile scores, the accuracy of real-time density forecasts is not significantly improved over 2007Q2-2019Q4.

Introduction of financial conditions

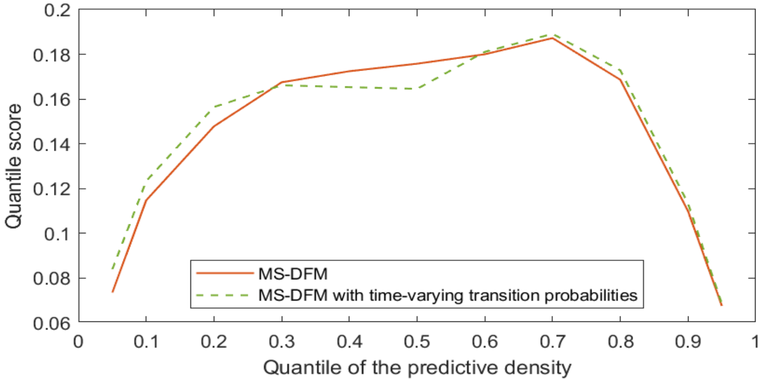
Figure: TV transition probability from the the Probit model



Estimation sample 1970M1-2019M12. Data vintage 2024M09.
Estimation based on 15000 draws of the Gibbs sampler, with the first 5000 discarded.

Introduction of financial conditions

Figure: Comparison of quantile scores (2007Q2-2019Q4).



Estimation sample starting in 1970M1. Real-time.
Forecast density based on 75000 draws of the Gibbs sampler, with the first 2500 discarded.

Treatment of Covid-19 period in the literature 1/2

Several papers suggest that Covid period data should be dropped

For instance:

- Lenza and Primiceri (JAE 2022): estimation of VAR model. Compare introduction of SV / dropping the observations of the Covid period.
  The 2nd method gives correct parameter estimates (but underestimates uncertainty).

- Maroz, Stock and Watson (WP 2021):compare DFM including a specific Covid factor / dropping the observations of the Covid period.
  The variations in the data are dominated by the Covid factor during the pandemic period $\rightarrow$ "these data should be ignored when studying non-Covid questions".

Treatment of Covid-19 period in the literature 2/2

- Schorfeide and Song (NBER WP, 2021): mixed frequency VAR. Exclude observations (first half of 2020).
- Baumeister and Hamilton (WP 2023) (granular IV estimation) : exclude Covid period.

Other papers propose models allowing to deal with outliers in order to keep Covid period. For instance:

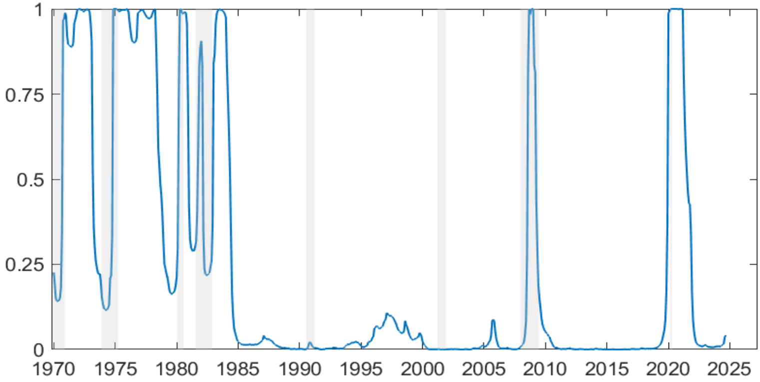
- Carriero et al. (Restat 2022): BVAR with SV and outliers with various specifications
- Antolin-Diaz et al. (JoE 2024): DFM with SV and outliers

Two approaches for the Covid 19 period

- We want to keep our parsimonious model.
- We want to keep 2 states for the mean and 2 states for the volatility.
- We thus propose two approaches:
  1. Estimate the factors and the probabilities of being in recession during the Covid period and after, but draw the parameters in the posterior distributions which are obtained when the model is estimated on 1970M1-2019M12.
  2. Skip the Covid period.
- In both cases we have, at this stage, considered that Covid period covered 2020 and 2021 (could probably be shortened)

First approach: frozen parameters distributions

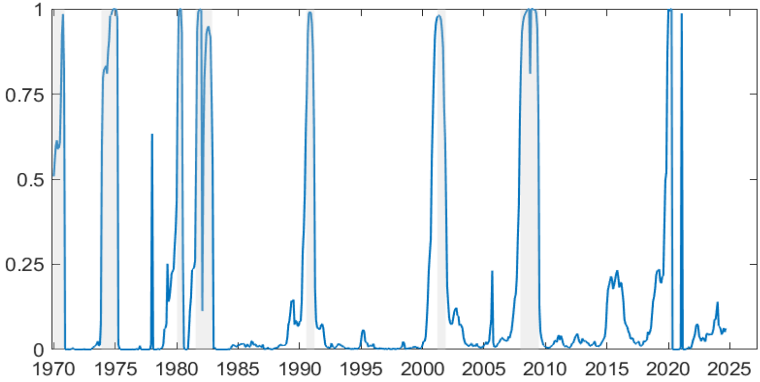
Figure: Probability of being in a high volatility regime



Data vintage 2024M09
Estimation based on 15000 draws of the Gibbs sampler, with the first 5000 discarded.

First approach: frozen parameters distributions

Figure: Probability of being in recession

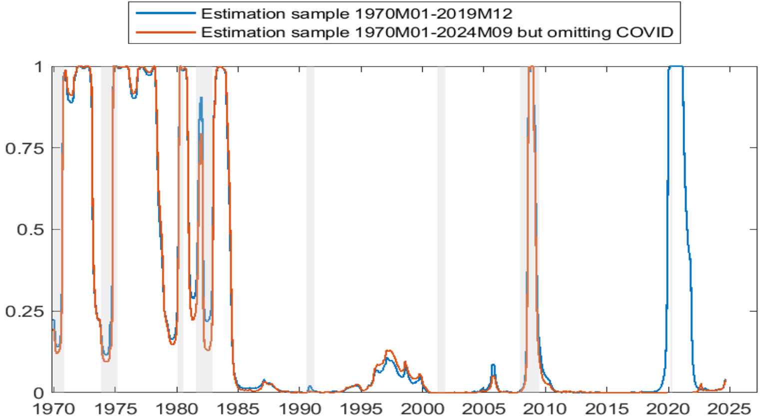


Data vintage 2024M09

Estimation based on 15000 draws of the Gibbs sampler, with the first 5000 discarded.

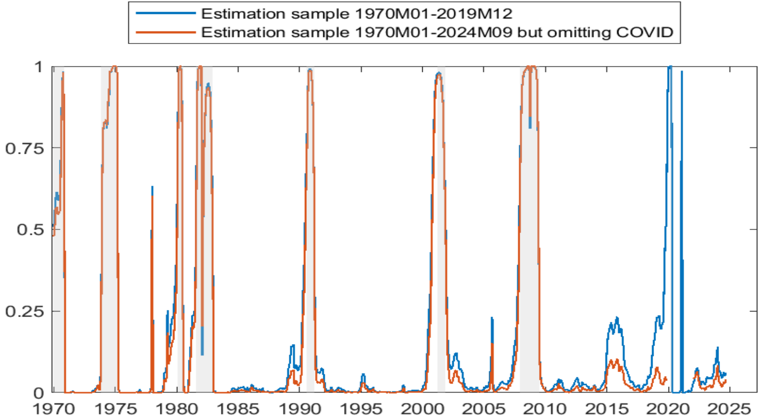
Second approach: skip the Covid period

Figure: Probability of being in a high volatility regime



Second approach: skip the Covid period

Figure: Probability of being in a recession



Conclusions and way forward 1/2

- We put forward a new extended MS-DFM allowing for both switches in intercept and volatility (MV-MS-DFM), as well as long-run time-varying GDP growth
- Introduction of MS volatility improves the detection of turning points during the Great Moderation and is supported by goodness-of-fit measures
- Our Extended MS-DFM captures in real-time turning points of the Great Recession with a lead of several months compared to NBER announcements
- The impact of volatility is stronger during recessions than during expansions and the conditional density forecasts are left-skewed. This result is in line with Adrian et al. (2019) introduction of Growth at Risk.

Conclusions and way forward 2/2

- The Great Recession is not the end of the Great Moderation, at least for the US economy
- Evidence of slowdown in long-run US GDP growth since 2000 (loss of about 1pp, half of this loss since the Great Recession).
- Our Bayesian estimation method allows to build real-time density forecasts. Those density forecasts provide a new evidence of the GaR phenomenon, but without any financial variable.
- Inclusion of financial variables though a probit specification seems to be a direction which could be fruitfully explored.
- We propose two approaches to deal with the Covid period

THANK YOU !