

High-Frequency Monitoring of Growth-at-Risk ¹

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¹The views expressed here are those of the authors and do not necessarily reflect those of the Banque de France

Plan of the talk

- 1 Motivations and Benchmark Growth-at-Risk
- 2 Our contribution
- 3 Econometric Modelling
- 4 Empirical Applications
- 5 Conclusions and Extensions

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- In our world of increasing uncertainties from various sources, efforts have been done to provide with a more accurate quantitative picture of **macro risks assessment**.
- This is what we do here, especially focusing on high-frequency monitoring of macro risks, in order to speed up the policy responses.

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- Idea: Do analogy with VaR (Value-at-Risk), risk measure well-known in finance, to assess macroeconomic risks (ie: Growth-at-Risk), in particular focusing on tail risks in the left part of the distribution
- $\text{GaR}(\tau \%)$ is the quantile at $\tau \%$ of the future GDP distribution
- This $\text{GaR}(\tau \%)$ can be computed on a regular basis, enabling to assess the evolution overtime of the macro risks

Growth-at-Risk

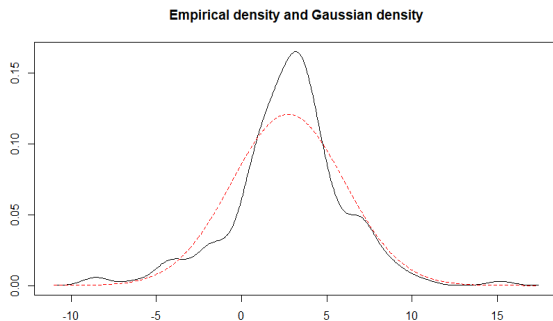
- Not an easy task as GDP growth distribution is unknown but presents various departures from Gaussianity (asymmetry / fat tails)
- If we assume a parametric distribution, a need for a more flexible distribution than the Gaussian, as the Generalized Skewed-Student distribution (Azzalini and Capitanio, 2003)
- Generalized Skewed-Student density function:

$$f(y; \mu, \sigma, \alpha, \nu) = \frac{2}{\sigma} t\left(\frac{y - \mu}{\sigma}; \nu\right) T\left(\alpha \frac{y - \mu}{\sigma} \sqrt{\frac{\nu + 1}{\nu + \left(\frac{y - \mu}{\sigma}\right)^2}}; \nu + 1\right), \quad (1)$$

where μ : location, σ : scale, ν : fatness and α : shape. $t(\cdot)$ and $T(\cdot)$ are (*pdf*) and (*cdf*) of standard Student

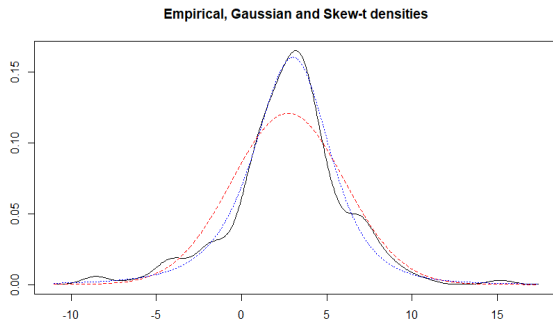
Growth-at-Risk

Unconditional distribution of US GDP growth rate



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- Thus the GaR can be seen as a measure of macroeconomic risks generated by financial markets conditions.
- Obviously, the GaR can be extended to various other conditioning sets considered as sources of macro risks.

Growth-at-Risk Methodology

- Assume we want to assess the joint effect of past GDP growth (y_{t-h}) and a given financial conditions indicator (FCI) (x_{t-h}), where h is the forecast horizon, on the current GDP growth (y_t)

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- Here the non-linearity is accounted for through a *quantile regression* of GDP on past FCI and past GDP

Growth-at-Risk Methodology

- Let's consider the following quantile regression with quarterly data:

$$y_t = \beta_1(\tau)y_{t-h} + \beta_2(\tau)x_{t-h} + \epsilon_t, \quad (2)$$

where parameters depend on the τ -th quantile of ϵ_t .

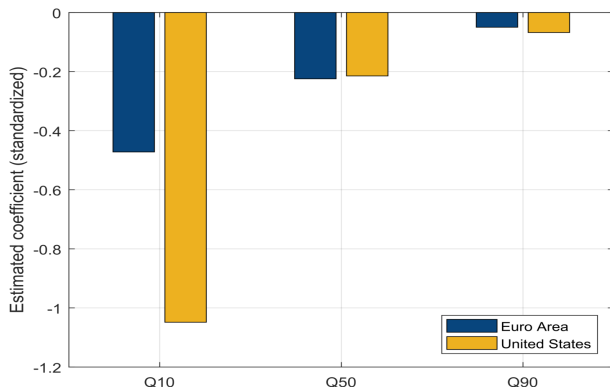
- $\hat{\beta}(\tau) = (\hat{\beta}_1(\tau), \hat{\beta}_2(\tau))$ are obtained by minimizing:

$$\sum_{t=1}^T \rho_{\tau}(y_t - \beta(\tau)' \mathbf{z}_{t-h}), \quad (3)$$

where $\mathbf{z}_{t-h} = (y_{t-h}, x_{t-h})'$, $\rho_{\tau}(u) = u(\tau - I(u < 0))$ is the loss function

Growth-at-Risk Methodology

Empirical results: Elasticity of GDP growth to quarterly FCIs for various quantiles (for both US and EA)



Growth-at-Risk Methodology

- Based on this quantile regression, predicted quantiles of GDP growth at T , conditional on information at $T - h$, are:

$$\hat{Q}_{y_{T|T-h}}(\tau|\mathbf{z}) = \hat{\beta}(\tau)' \mathbf{z}_{T-h}. \quad (4)$$

- In a second step we fit a generalized Skewed-Student distribution to those conditional quantiles, the 4 parameters are estimated through a quantile matching approach:

$$\min_{\mu, \sigma, \alpha, \nu} \sum_{\tau} \left[\hat{Q}_{y_{T|T-h}}(\tau|\mathbf{z}) - F^{-1}(\tau; \mu, \sigma, \alpha, \nu) \right]^2, \quad (5)$$

where $F^{-1}(\cdot)$ is the inverse cumulative Skewed-Student distribution.

Growth-at-Risk Methodology

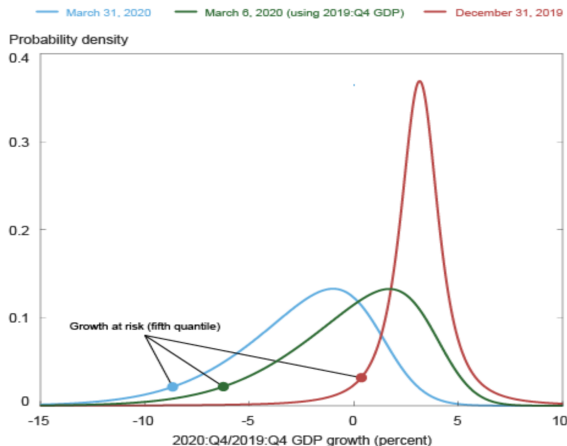
- *In fine*, we focus here on the GaR(10%), given by:

$$Q_{y_T|T-h}^*(\tau = 0.10|\mathbf{z}) := F^{-1}(\tau = 0.10; \hat{\mu}, \hat{\sigma}, \hat{\alpha}, \hat{\nu}). \quad (6)$$

- This can be interpreted as the expected value of future GDP at 10% probability, stemming from the quantile function conditional to past GDP and past financial conditions.

Growth-at-Risk: Example

US economy during the Covid crisis (source: Adrian et al., 2020, LibertyStreet Blog, NY Fed)



Our contribution

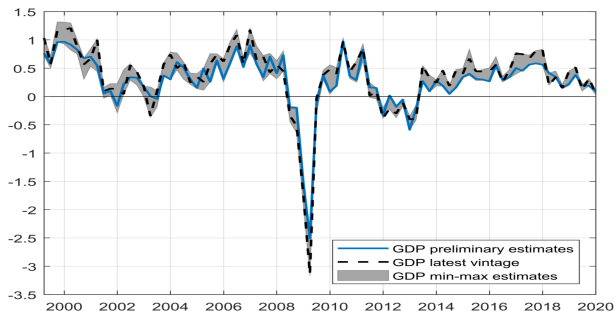
- We extend the GaR approach of Adrian *et al.* (2019) by accounting for the high-frequency nature of the FCIs through a Mixed Data Sampling (MIDAS) approach.
- We focus on the euro area and we consider two daily financial condition indexes (BdF FCI and ECB CISS) for which we provide an approach to optimally combine those 2 indexes.
- We focus on the 10th quantile of the conditional predictive distribution of EA GDP. We build an indicator of financial downward risks to *current* real activity: **GaR(10%)**.

Our contribution

- We hence put forward an empirical model based on MIDAS quantile regressions and a real-time framework that provides a useful tool for policymakers for:
 - ① **Daily monitoring** of financial risks to real activity.
 - ② **Nowcasting** measure of GDP tail risks (Carriero, Clark and Marcellino, 2020).
- We provide 3 various applications highlighting the use of our tool in real-time for policy-makers.

Data: Vintages of GDP

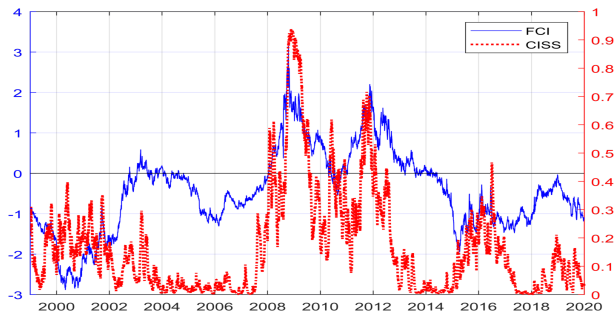
- We collected EA GDP vintages collected in a data trapezoid spanning from 1999Q1 to 2019Q4. Multiple releases per quarter (2 to 4 overtime).



Data: Euro area daily FCIs

- We consider two daily FCIs available for EA:
 - ① BdF daily Financial Conditions Index (**FCI**; Petronevich and Sahuc, 2019). Based on PCA over 18 variables. Useful for exploring macro-financial linkages.
 - ② ECB daily Composite Indicator of Systemic Stress (**CISS**; Holló et al., 2012). Time-Varying aggregation of market-specific indexes: FX, equity, money, bond, financial intermediaries. Financial stress indicator, designed to react more to systemic fragility in financial markets.
- Both indexes measure different objects, but present some correlation: we have to think about how to combine them.

Data: Euro area daily FCIs



Quantile MIDAS regression

- FCI $x_t^{(d)}$ is available on a daily basis, i.e. without delay, and is observed about $d = 60$ times between quarters $t - 1$ and t
- Our high-frequency real-time GaR measure relates current GDP growth to (i) past GDP and to (ii) past *and* current financial conditions, up to the latest available daily observation.
- MIDAS-quantile regression (MIDAS-QR) for $h = 1$:

$$y_t = \beta_1(\tau)y_{t-1} + \beta_2(\tau) \sum_{c=0}^{C-1} \tilde{B}(c; \theta(\tau)) L^{c/d} x_{t-h_d}^{(d)} + \epsilon_t \quad (7)$$

where $\tilde{B}(c; \theta(\tau))$ is a weighting function (normalized to sum up to 1), which depends on a vector of parameters $\theta(\tau)$ and a lag order $c = 0, \dots, C - 1$.

Quantile MIDAS regression

- MIDAS function: un-normalized Almon lag polynomial, as it is linear and parsimonious, as in Mogliani and Simoni (2020):

$$\tilde{B}(c; \boldsymbol{\theta}(\tau)) = \sum_{i=0}^p \theta_i(\tau) c^i,$$

where $\boldsymbol{\theta}(\tau) := (\theta_0(\tau), \theta_1(\tau), \dots, \theta_p(\tau))'$,

- MIDAS-QR boils down to:

$$y_t = \beta_1(\tau) y_{t-1} + \boldsymbol{\theta}(\tau)' \tilde{\mathbf{x}}_{t-h_d}^{(d)} + \epsilon_t, \quad (8)$$

where $\tilde{\mathbf{x}}_t^{(d)} := \mathbf{Q} \mathbf{x}_t^{(d)}$ is a $(p+1) \times 1$ vector of linear combinations of the high-frequency FCI,

Bayesian estimation

- The original minimization problem (Koenker and Bassett, 1978) is equivalent to a maximization problem under Asymmetric Laplace error Distribution (ALD; Yu and Moyeed, 2001).
- Kozumi and Kobayashi (2011) show that the ALD can be seen as a mixture of an exponential and a scaled normal distributions,
$$\epsilon = \xi_1 \sigma \nu + \xi_2 \sigma \sqrt{\nu} \omega$$
- Conditional likelihood function is Gaussian. Under standard conditionally Normal prior, Gibbs sampling can be efficiently implemented to draw from the full conditional posteriors of the parameters.

Combining densities

- FCI and the CISS highly correlated. Risks of multicollinearity in the model (poor inference/predictive results)
 \Rightarrow **density forecast combination approach**
- We put forward a way to combine densities by accounting for their ability to reproduce observed quantiles
- Compute $Q_{i, y_{T|T-h_d}}^*(\tau | \mathbf{X}_i)$ for \mathbf{X}_i including either the FCI ($i = 1$) or the CISS ($i = 2$).
- The Quantile Weighted Probability Score (Gneiting and Ranjan, 2011) is computed for each model $i = \{1, 2\}$.

Combining densities

- Combination weights $\omega_{i,T-h_d}$ computed recursively using a discounted QWPS combinations method:

$$w_{i,T-h_d} = \sum_{j=T_0}^{T_\ell} \delta^{T_\ell-j} \text{QWPS}_{i,j} \qquad \omega_{i,T-h_d} = \frac{w_{i,T-h_d}}{\sum_{i=1,2} w_{i,T-h_d}}$$

- Combined conditional predictive quantile function is computed:

$$Q_{c,y_{T|T-h_d}}^*(\tau|\mathbf{X}) = \sum_{i=1,2} \omega_{i,T-h_d} \times Q_{i,y_{T|T-h_d}}^*(\tau|\mathbf{X}_i)$$

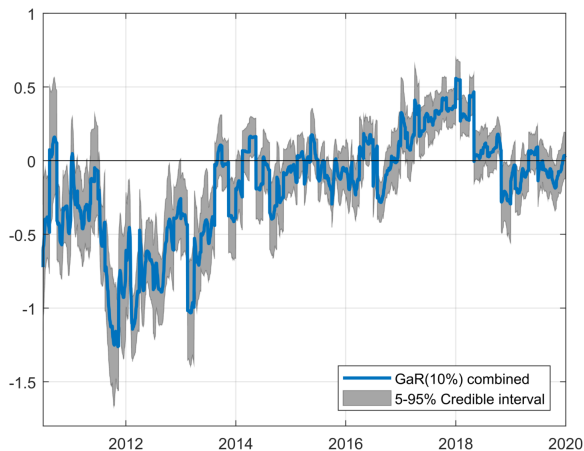
The algorithm

- 1 For each financial index $i = 1, 2$, Bayesian estimation of MIDAS-QR model and the τ -th conditional predictive quantile function of $y_{T|T-h_d}$, conditional on sample information available up to $T - h_d$, for $h_d = 0, 1/d, 2/d, \dots$ (in practice, $d = 60$).
- 2 Fit Skewed t -distribution to $\hat{Q}_{y_{T|T-h_d}}(\tau|X)$, and get $Q_{y_{T|T-h_d}}^*(\tau|X)$.
- 3 Gather $Q_{y_{T|T-h_d}}^*(\tau = 0.10|\mathbf{X}_i)$
- 4 Combined conditional predictive quantile function is computed:

$$Q_{c,y_{T|T-h_d}}^*(\tau|\mathbf{X}) = \sum_i \omega_{i,T-h_d} \times Q_{i,y_{T|T-h_d}}^*(\tau|\mathbf{X}_i)$$

\Rightarrow Real-time combined high-frequency measure of downward financial risks to real activity – GaR(10%)

The daily combined GaR(10%) (2010Q3-2019Q4)



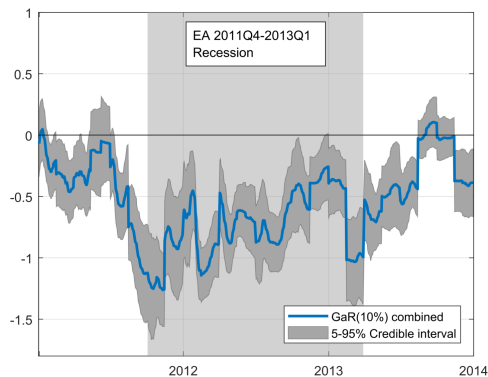
Three applications (main results)

- **The 2011Q4-2013Q1 EA recession:** the GaR(10%) quickly and significantly captures a risk of recession in S2 2011
- **Nowcasting (2010Q3-2019Q4):** GDP growth nowcasting using results for all quantiles; outperforms its competitors for relatively short forecast horizons.
- **Covid-19 crisis:** Swift responses from the GaR(10%), but do not fully recover the amplitude of the pandemic shock

Application 1 - The 2011Q4-2013Q1 EA recession

- In the wake of the Great Recession, double-dip in EA in 2011-13 related to an increase in sovereign spreads in peripheral countries and tensions in the banking system
- Severe deterioration in global financial markets in the course of July and early August 2011 due to concerns regarding the sovereign debt crisis and high levels of uncertainty about the sustainability of public finances in many Euro area countries.
- The GaR(10%) quickly captures a risk of future recession (a turning point in the business cycle in 2011Q4 is officially announced by the CEPR in February 2012).

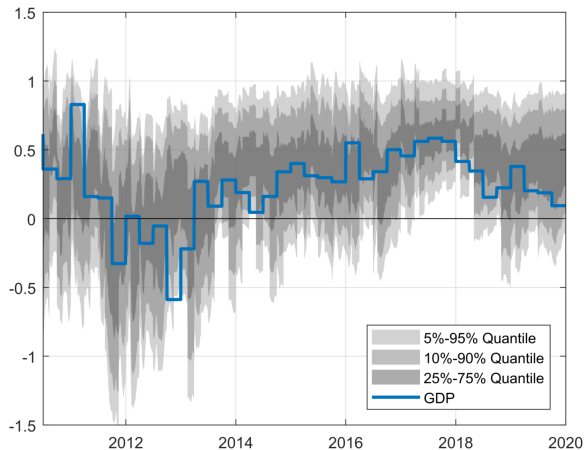
Application 1 - The 2011Q4-2013Q1 EA recession



Application 2 - Nowcasting (2010Q3-2019Q4)

- From the Bayesian estimation, we get all the quantiles of the distribution, so it's tempting to use mean/median as an estimate of GDP growth.
- We compare the forecasts for various horizons wrt a simple Bayesian AR(1) as well as with 2 other competitors:
 - ① BMIDAS: A Bayesian MIDAS that accounts only for mixed-frequencies but in a linear setting
 - ② BQAR(1): A Bayesian Quantile AR(1) that accounts for non-linearities through quantile regression, but on a quarterly basis

Application 2 - Nowcasting (2010Q3-2019Q4)



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- Density forecasts are compared based on 4 various criteria:
 - 1 LS: average Log-Score differentials
 - 2 CRPS: average Continuous Ranking Probability Score ratios
 - 3 QWPS: average Quantile-Weighted Probability Score ratios
 - 4 QS: average Quantile Score ratios
- For CRPS, QWPS and QS criteria, values less than one indicate that our combined model outperforms the benchmark BAR.
- For the LS criterion, positive values indicate that our model produces more accurate density forecasts than the benchmark BAR.

Application 2 - Nowcasting (2010Q3-2019Q4)

Table: Out-of-sample results: relative accuracy of density forecasts

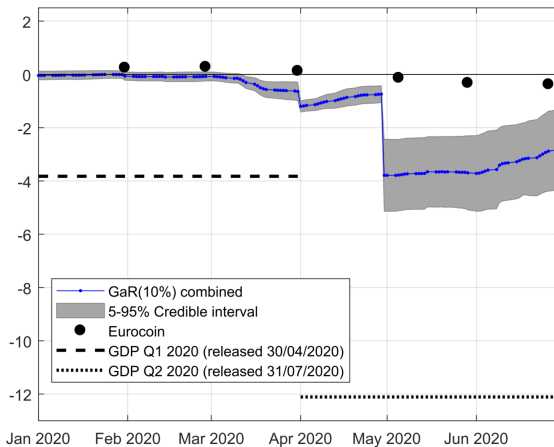
| h_d | BMIDAS-QR | | | | BMIDAS | | | | BQAR(1) | | | |
|-------|-------------|-------------|-------------|-------------|--------|------|------|----------|-------------|-------------|-------------|-------------|
| | LS | CRPS | QWPS | QS(0.10) | LS | CRPS | QWPS | QS(0.10) | LS | CRPS | QWPS | QS(0.10) |
| 0 | 0.22 | 0.91 | 0.86 | 0.79 | -0.03 | 1.06 | 0.97 | 0.95 | 0.13 | 0.94 | 0.97 | 0.95 |
| 10 | 0.22 | 0.90 | 0.84 | 0.77 | -0.03 | 1.04 | 0.96 | 0.96 | 0.13 | 0.94 | 0.97 | 0.95 |
| 20 | 0.24 | 0.88 | 0.83 | 0.77 | -0.02 | 1.03 | 0.96 | 0.95 | 0.13 | 0.94 | 0.97 | 0.94 |
| 40 | 0.10 | 1.01 | 1.01 | 0.88 | -0.05 | 1.14 | 1.09 | 0.94 | 0.13 | 0.95 | 0.96 | 0.87 |
| 60 | 0.10 | 0.99 | 1.00 | 0.85 | -0.05 | 1.11 | 1.07 | 0.94 | 0.14 | 0.94 | 0.95 | 0.87 |

Notes: LS, CRPS, QWPS, and QS denote respectively the log-Score, the Continuously Ranked Probability Score, the Quantile Weighted Probability Score, and the Quantile Score (at $\tau = 0.10$), in relative terms with respect to the AR(1) benchmark. Bold-blue values denote the best outcomes for each forecast horizon h_d .

Application 3 - The daily GaR(10%) and the COVID-19 episode

- First significant decline in the GaR(10%) around mid-March: *i)* WHO announcement recognizing the Covid-19 global pandemic (March 11), and *ii)* start of stringent lockdown measures within euro area countries.
- Second drop at the end of April (integration of GDP growth for 2020q1 into the model).
- More rapid answer to the crisis than the EuroCoin index aiming at nowcasting EA GDP in real-time

Application 3 - The daily GaR(10%) and the COVID-19 episode



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- Clear timely signals of increasing risks on economic activity, but of limited amplitude
- This crisis is a mix of supply and demand shocks, amplified by an uncertainty shock, but (so far) moderate financial shock (due to swift and strong monetary policy reaction by the ECB and other big CBs).
- It will be useful to track the GaR(10%) in upcoming months as the number of company defaults tends to increase, especially when policy support will fade away. It is likely to affect negatively financial conditions.

Conclusions

- We provided a real-time high-frequency tool for monitoring downside financial risks to GDP growth of the Euro area.
- We provided examples of the effectiveness of this policy tool in terms of real-time macroeconomic analysis.
- During the Covid-19 episode, the EA daily GaR(10%) has provided a timely indication of tail risks on GDP, but *this time is different* as all the macro risks don't come from the financial sector.

Extension 1: GaR beyond FCIs

- Idea: FCIs are not systematically relevant for macro risks
⇒ macro variables could also be useful

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- Idea: FCIs are not systematically relevant for macro risks
⇒ macro variables could also be useful
- Recent papers by Adrian et al. (2019) or Caldara et al. (2020) highlight the dependence between financial and real areas, generally non-linear
- But paper by Plagborg-Moller, Reichlin, Ricco, Hasenzagl (2020) point out that financial variables provide only limited information as regards future US recessions
- Story behind: not all macro risks are generated by financial shocks (GFC vs Covid-19)

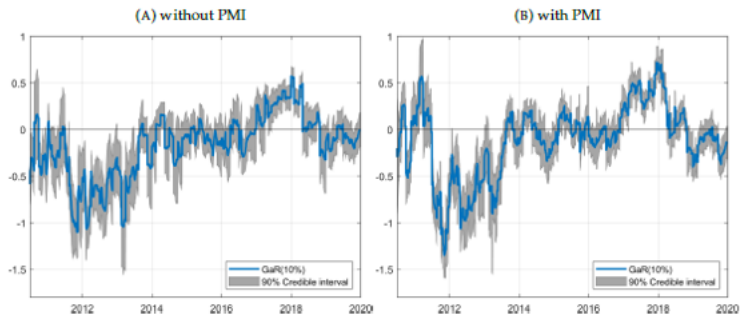
Extension 1: GaR beyond FCIs

- What we do: simultaneously account for daily FCIs and monthly forward-looking macro variables (eg: PMI)
- MIDAS equation with 2 high frequencies:

$$y_t = \beta_1(\tau)y_{t-1} + \theta(\tau)\tilde{\mathbf{x}}_{t-h_d}^{(d)} + \phi(\tau)\tilde{\mathbf{x}}_{t-h_m}^{(m)} + \epsilon_t, \quad (9)$$

- New results tend to show similar results for the GaR(10%), with more sensitivity to the cycle (more accurate results for mean forecasting, see Figure next slide), meaning that the FCI information remains important when we include a forward-looking macro variable

Extension 1: GaR beyond FCIs



Extension 2: Large dimension GaR

- Idea: Multi-dimensional coverage of future macro risks

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- Other sources of information of macro risks are credit quantities, housing prices, exchange rates, short-term external debt, current account ... They can reflect possible increasing imbalances turning into macro risks
- Empirical issue: How to deal with large datasets against the GaR background?
 - Dimension reduction techniques, such as DFM (Plagborg-Moller et al., 2020) , Lasso (Mitchell et al, 2020) or Group-Lasso (Mogliani and Simoni, 2020), see also Manzan (2015)
 - Estimate GaR for each time series, then combine densities. Optimal weights? Large literature ...
Quantile aggregation is a simple way to combine distributions (Busetti, 2017)

Extension 3: X-at-Risk

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- Inflation at Risk: High-frequency Inflationary/Deflationary pressures can be assessed against this background, by using daily market expectations of break-even inflation rate
Recent works on IaR by Ghysels, Iana and Striaukas (2018); Lopez-Salido and Loria (2020)
- GDP components at Risk: Business investment, Consumption, Exports ... using FCI or other relevant high-frequency variables