Auto-Regressive Dynamic Linear models

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Plan

Introduction

- Introduce dynamics into the linear regression model, especially useful for macroeconomic forecasting
- past values of the dependent variable, but also of exogeneous variables
- What type of information can be useful?
  - Coincident macro indicators: hard data (IPI, retail sales, consumption ...)
  - Coincident/Leading indicators: opinion surveys (households, companies), expectations
  - Leading indicators: financial variables (credit spreads, volatility, asset prices)
  - Composite indicators (OECD, US CLI ...)

Correlations

How to measure the relationship between Y_t and X_t?

Basic tool: contemporeanous correlation

$$\rho(X_t, Y_t) = \frac{cov(X_t, Y_t)}{\sigma_X \sigma_Y}$$

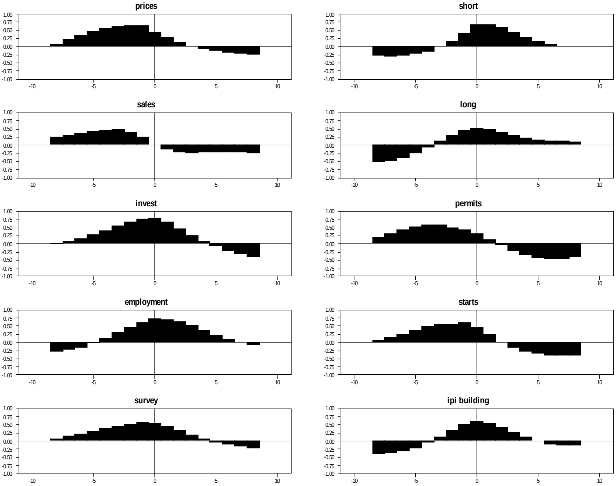
Alternative: cross-correlation at a given lag k

$$\rho(X_t, Y_{t-k}) = \frac{cov(X_t, Y_{t-k})}{\sigma_X \sigma_Y}$$

Example of cross-correlation: French housing market

- Objective: Etablish cyclical relationships between a set of macro and housing variables using correlation analysis
- Selected variables 1980q1 - 2009q2:
- Macro: GDP, Household investment, Employment in construction, IPI in construction
- Housing: Real prices, Sales, Permits, Starts, Survey by Property Developers
- Finance: Long (Gov. bonds 10 years) and Short (Euribor 3-months) interest rates

Cross-correlation with GDP



Cross-correlation Analysis

	GDP	Prices	Sales	Invest.	Employ.	Survey	Short	Long	Permits	Starts
GDP		0.64	0.48	0.80	0.72	0.58	0.68	-0.52	0.58	0.62
Prices	-2		0.36	0.70	0.71	0.50	0.54	0.44	0.61	0.62
Sales	-3	-2		0.60	0.27	0.35	-0.37	-0.49	0.44	0.61
Investment	0	+1	+3		0.69	0.53	0.50	0.54	0.63	0.70
Employment	0	+2	+7	+1		0.55	0.56	0.64	0.52	0.47
Survey	-1	+1	+3	0	-2		0.37	-0.41	0.42	0.44
Short rate	0	+3	0	+1	0	+2		0.59	0.44	0.43
Long rate	-8	+3	-1	+3	0	-7	-1		-0.48	-0.47
Permits	-4	0	+1	-3	-4	-2	-5	+6		0.69
Starts	-1	0	+2	-1	-4	-1	-4	+5	+1	
IPI	0	+2	-2	0	0	+2	-1	+8	+4	+3

Table: Highest cross-correlation coefficients among all leads and lags (upper diagonal, lags in parenthesis) and leads/leags (lower diagonal), from 1980 Q1 to 2009 Q2. A negative number indicates that the series in row leads the series in column with an advance equal to this number, and conversely.

Cross-correlation with GDP

- A leading pattern in housing variables (Real prices, Sales, Permits and Starts)
- Residential investement is strongly related to the economic cycle, in coincident manner
- Employent and IPI in construction are coincident with GDP
- Short rate (3m): a positive correlation with a short delay (0-1 quarters)
- Long rate (10y): a negative correlation with lead of 2 years.

ARDL models

How to integrate dynamics into a mutivariate linear model?

Definition

$$Y_t = \alpha + \sum_{j=0}^{m} \beta_j' X_{t-j} + \sum_{j=1}^{p} \phi_j Y_{t-j} + \varepsilon_t, \tag{1}$$

where:
X_t is the k-vector of explanatory variables $(X_{1t}, \dots, X_{kt})'$,
m is the lag of the explanatory variables,
p is the AR order
ε_t strong WN,
for a given lag j, $\beta_j = (\beta_j^1, \dots, \beta_j^k)'$ is the coefficient vector for explanatory variables of length k.

ARDL models

- The model specification is generally carried out using information criteria such as AIC or BIC.
- Note that m is not necessarily equal for all X_j, can be m_j, for $j = 1, \ldots, k$
- The $mk + p + 1$ parameters of the model can be estimated by ordinary least-squares

Forecasting using ARDL models

Assume $k = 1$ explanatory variable, $m = 1$, $p = 1$, $h = 1$:

$$Y_t = \alpha + \beta X_t + \phi Y_{t-1} + \varepsilon_t \tag{2}$$

The 1-step ahead forecast is the conditional expectation given by:

$$\hat{Y}_t(1) = E(Y_{t+1}|I_t) = \hat{\alpha} + \hat{\beta} E(X_{t+1}|I_t) + \hat{\phi} Y_t \tag{3}$$

Major empirical issue in real-time
In general, $E(X_{t+1}|I_t)$ is unknown: How to estimate it?

Forecasting using ARDL models

3 approaches to forecast using ARDL models:

1. Iterative forecasting: Conditional forecasting of explanatory variables
2. Scenario forecasting: Judgemental forecasting of explanatory variables
3. Direct forecasting: a specific regression for each horizon h

1/ Iterative forecasting based on ARDL

How to compute $E(X_{t+1}|I_t)$? Use of auxiliary models.

Example: Assume that (X_t) follows an AR(1):

$$X_t = \phi X_{t-1} + u_t$$

$$E(X_{t+1}|I_t) = \hat{\phi} X_t$$

2/ Scenario forecasting based on ARDL

Example of 3 scenarii for $E(X_{t+1}|I_t)$:

1. negative growth of -3% (X^{-}_{t+1})
2. stability : 0% (X^{0}_{t+1})
3. positive growth of +2% (X^{+}_{t+1})

$$\hat{Y}^{-}_{t}(1) = \hat{\alpha} + \hat{\beta} X^{-}_{t+1} + \hat{\phi} Y_t \tag{4}$$

$$\hat{Y}^{0}_{t}(1) = \hat{\alpha} + \hat{\beta} X^{0}_{t+1} + \hat{\phi} Y_t \tag{5}$$

$$\hat{Y}^{+}_{t}(1) = \hat{\alpha} + \hat{\beta} X^{+}_{t+1} + \hat{\phi} Y_t \tag{6}$$

3/ Direct forecasting based on ARDL

Let's consider the following ARDL model for a specific horizon $h > 0$

$$Y_{t+h} = \alpha_h + \sum_{j=0}^{m} \beta'_{hj} X_{t-j} + \sum_{j=0}^{p} \phi_{hj} Y_{t-j} + \varepsilon_{t+h}, \tag{7}$$

where, for a given lag j, $\beta_{hj} = (\beta_{hj}^{1}, \ldots, \beta_{hj}^{k})'$ is the coefficient vector for exogeneous variables of length k, depending on h. For each h there is a specific equation.

The h-step-ahead forecast is thus given by:

$$\hat{Y}_t(h) = \hat{\alpha}_h + \sum_{j=0}^{m} \hat{\beta}'_{hj} X_{t-j} + \sum_{j=0}^{p} \hat{\phi}_{hj} Y_{t-j}. \tag{8}$$

References related GDP forecasting based on financial variables

US data:

Estrella and Hardouvelis (1991), Hamilton and Kim (2002), Estrella et al. (2003) or Giacomini and Rossi (2006).

Gilchrist and Zakrajsek (2012): a new credit spread index to predict US GDP growth.

Estrella and Mishkin (1997): usefulness of various term spreads and monetary variables for the US GDP

Euro area:

Andersson and d'Agostino (2008) use sectoral stock prices to predict the euro area GDP.

Duarte et al. (2005) spread between 10-year sovereign yeld and the 3-month interbank rate

Example: Forecasting GDP

- A simple example of forecasting with the following specification (also known as the *Leading Indicator* model):

$$Y_t = \alpha + \beta X_{t-h} + \varepsilon_t$$

where we shift back the regressors from t to $t - h$

- Generally $h = 1$

- Issue: find the leading regressors with the convenient lead h

Example: Taylor rule

Extended regression for central bank interest by accounting for persistence in the interest rate (Smoothed/Inertial Taylor rule):

$$r_t = \rho r_{t-1} + \alpha + \beta g_t + \gamma(\pi_t - \pi*) + \varepsilon_t$$

where: ρ controls the persistence (generally estimated around 0.85)

Variant of the extended regression (cf. Atlanta Fed web site):

$$r_t = \rho r_{t-1} + (1-\rho)\{\alpha + \beta g_t + \gamma(\pi_t - \pi*)\} + \varepsilon_t$$

Calibration of a Smoothed Taylor rule